Hence,

$$t_1 + t_2 + t_3 = 25 \text{ min. } 40.9 \text{ sec.}$$

Finally, the numerical data satisfy the following check equations, obtained by projection of the path on the coördinate axes,

$$l_1 \cos \phi + l_2 \cos (\phi + i - r) + l_3 \cos (\phi + 2i - 2r) = 1,$$

$$l_1 \sin \phi + l_2 \sin (\phi + i - r) + l_3 \sin (\phi + 2i - 2r) = 1.$$

II. Solution by A. H. Holmes, Brunswick, Maine.

Let $ABCD$ (Fig. 2) be the section of land, 1 mile square, A being the southeast corner, B the northeast corner, C the northwest corner, and D the southwest corner. On DC, take $DE = 120$ rods or $\frac{3}{8}$ of a mile and draw EF perpendicular to AB at F. On EF, take $EO = 1/\pi$ of a mile to point O.

With O as a center and EO as radius describe a circle. Draw AO and CO. Let AG be the course to the circumference of track, GH the course inside the track, and HC the remainder of the course to the northwest corner C.

Draw OK perpendicular to GH at K. The ratio of GH to $AG + CH$ is the greatest when OK bisects the angle AOC. Therefore, for a minimum, the angles AOG and COH are equal.

Put $AG = x$, $CH = y$, $GH = z$, $AO = a$, $CO = b$, $1/\pi = r$, $AOG = COH = \theta$, and $AOC = 2m$.

Then

$$\frac{x+y}{3} + \frac{z}{4} = \text{minimum}.$$

In $\triangle\, AGO$ and CHO

$$\cos \theta = \frac{a^2 + r^2 - x^2}{2ar}, \qquad \cos \theta = \frac{b^2 + r^2 - y^2}{2br}.$$

Hence $x = \sqrt{a^2 + r^2 - 2ar \cos \theta}$, $y = \sqrt{b^2 + r^2 - 2br \cos \theta}$, and $z = 2r \sin (m - \theta)$ in which $a = .778027$, $b = .701390$, $r = .318310$, and $m = 72° \; 54' \; 9''$.

Multiplying x and y by 4 and z by 3, reducing and differentiating,

$$\frac{2a \sin \theta}{\sqrt{a^2 + r^2 - 2ar \cos \theta}} + \frac{2b \sin \theta}{\sqrt{b^2 + r^2 - 2br \cos \theta}} = 3 \cos (m - \theta).$$

Whence, $\theta = 12° \; 33' \; 9''$.

Knowing θ, the required course in distances and directions is easily found.

From A to G, N. 52° 45′ 48″ W. 151.17 rods; G to H, N. 44° 5′ 13″ W. 177.03 rods; H to C, N. 37° 1′ 51″ W. 126.96 rods, the length of the whole course being 455.16 rods, and made in the minimum time of 25 minutes 40.8 seconds.

MECHANICS.

303. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

A pile driver weighing 500 pounds falls through 10 feet and drives a pile weighing 400 pounds 3 inches into the ground. Show that the average force of the blow is $11{,}111\frac{1}{9}$ pounds.

Solution by W. H. Williams, Oakland, California.

Let m = mass of driver in pounds, m' = mass of pile in pounds, v = velocity of driver at instant of impact in feet per second, h = height of fall of driver in feet, d = distance pile is driven into ground in feet, r = average retardation of pile and driver after impact in feet per second per second, t = time from impact until rest in seconds, and g = acceleration due to gravity in feet per second per second, F = average force of blow.

Then mv = momentum of driver at impact = momentum of driver + pile immediately after impact; $mv/(m + m')$ = common velocity of driver and pile immediately after impact.

Hence,

$$\frac{mv}{(m+m')r} = t, \qquad \text{and} \qquad d = \tfrac{1}{2}rt^2 = \tfrac{1}{2}r\frac{m^2v^2}{(m+m')^2r^2} = \tfrac{1}{2}\frac{m^2v^2}{(m+m')^2r},$$

or

$$r = \tfrac{1}{2}\,\frac{m^2v^2}{(m+m')^2d}.$$

But $v^2 = 2gh$, and hence,

$$r = \frac{m^2gh}{(m+m')^2d}.$$

Finally,

$$F = (m+m')r = \frac{m^2gh}{(m+m')d}\text{ poundals} = \frac{m^2h}{(m+m')d}\text{ pounds.}$$

Substituting values,

$$F = \frac{500 \times 500 \times 10}{(500+400) \times \frac{1}{4}} = 11{,}111\tfrac{1}{9}\text{ lbs.}$$

Also solved by A. M. Harding and J. A. Caparo. Erroneously solved by Herbert N. Carleton.

305. Proposed by B. J. BROWN, Victor, Colorado.

A particle is to be projected so as to graze the top of a wall h feet high, at a distance of a feet from the point of projection, and to strike the ground at a distance b feet from the foot of the wall. Find the velocity of projection, and the inclination of the path to the horizontal, at the ground and at the top of the wall. I. C. S. 1903.

Solution by Emma M. Gibson, Assistant at Drury College.

The equation of the path of the particle is

$$y = x\tan\alpha - \tfrac{1}{2}g\,\frac{x^2}{\nu_0^2\cos^2\alpha},$$

where ν_0 is the initial velocity and α the angle of projection.

Knowing the points (a, h), $(a+b, 0)$ to be on the curve the following satisfied relations exist:

$$h = a\tan\alpha - \tfrac{1}{2}g\,\frac{a^2}{\nu_0^2\cos^2\alpha} \qquad (1)$$

and

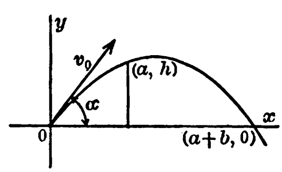
$$(a+b)\tan\alpha = \tfrac{1}{2}g\,\frac{(a+b)^2}{\nu_0^2\cos^2\alpha} = 0. \qquad (2)$$

From equation (2),

$$\nu_0^2 = \frac{(a+b)g}{2\sin\alpha\cos\alpha}, \qquad \text{or} \qquad \nu_0 = \sqrt{\frac{(a+b)g}{\sin 2\alpha}}.$$

Substituting this value of ν_0 in (1) and solving for $\tan\alpha$,

$$\tan\alpha = \frac{(a+b)h}{ab}.$$



Hence,

$$\alpha = \tan^{-1}\left[\frac{(a+b)h}{ab}\right],$$

the inclination of the path at the ground.

The equation of the tangent to the curve at any point (x_1, y_1) is

$$\frac{y+y_1}{2} = \frac{x+x_1}{2}\tan\alpha - \tfrac{1}{2}g\,\frac{x_1x}{\nu_0^2\cos^2\alpha}.$$

At (a, h) the tangent becomes

$$y + h = (x+a)\tan\alpha - \frac{agx}{\nu_0^2\cos\alpha} = \left[\frac{\nu_0^2\sin\alpha\cos\alpha - ag}{\nu_0^2\cos^2\alpha}\right]x + a\tan\alpha.$$

The inclination of this line,

$$\tan^{-1}\left[\frac{\nu_0^2\sin\alpha\cos\alpha - ag}{\nu^2\cos^2\alpha}\right] = \tan^{-1}\left[\left(\frac{b-a}{ab}\right)h\right],$$

is the inclination of the path to the horizontal at the top of the wall.